

Diffusion Limited Aggregation

Introduction

Diffusion Limited Aggregation (DLA) is one of the most puzzling models in probability and statistical physics. There are many thousands physics papers discussing this model and only one mathematical theorem.

Originally it was proposed by Sander and Witten as a model for irreversible colloidal aggregation but it was soon realized that it is widely applicable. In particular this model describes mineral deposition (see figure). In this model one starts with a single particle at the origin and then sends particle from infinity one by one. Particles perform random walk on the lattice (or Brownian motion in the off-lattice case) until they hit the aggregation. When they hit the aggregation, they stick to it and the next particle is sent from infinity.

There are many questions about long time behaviour of DLA: What is the growth rate? Is it a regular fractal? What is the Hausdorff dimension? Are there deep fjords? Is there a non-trivial scaling limit? Is there difference between lattice and off-lattice models?

These and many other questions were addressed by many physics papers and there were many computer simulations as well as real experiments. The only mathematical theorem about DLA is due to Kesten who proved that the diameter of the aggregation after N steps is $O(N^{2/3})$.



Manganese deposition on a limestone

Project

There are many computer experiments that one can perform. The starting point will be simulations of DLA and reproducing results about the growth rate of DLA. After that we can go into several possible directions: large scale simulations, dependence of the shape on the lattice structure, study of stationary distribution on the flat torus etc.

Prerequisites

There are no prerequisites since the model is very simple to formulate, it just uses simple random walk on the lattice. On the other hand, the model is very difficult to analyse, so there is almost no mathematical literature on the subject.

Reading

There is not much to read about DLA. The model was originally described in Witten and Sander, *Diffusion-Limited Aggregation, a Kinetic Critical Phenomenon*, Phys. Rev. Lett. vol. 47, **19**, 1981

There is a section about DLA in Lawler, Gregory F. *Intersections of random walks*. Probability and its Applications. Birkhuser Boston, Inc., Boston, MA, 1991